Duration Measures
1) Macaulay Duration is the time weighted present value of the bond's future cash flows divided by the bond's price. Think about this as the average time it takes to recover your initial investment through receipt of the cash flows.

Exam tip:
I really doubt you will have to calculate Macaulay duration on the exam...and even if it does show up - SKIP IT! Do not waste your precious time trying to get this correct...after you have completed the exam and got all your easy marks, then go back and spend the time to calculate it, but under no circumstances are you to waste 10 minutes on this as you go through the exam on your first pass.

Note:
The Macaulay duration of a zero coupon bond is equal to the bond's remaining time to maturity and the Macaulay duration of any coupon paying bond will always be less than the bond's remaining time to maturity.

2) Modified duration is used to estimate the approximate percentage change in bond value when yields change.

$$\text{Modified duration} = \frac{\text{Macaulay Duration}}{1 + (YTM/n)}$$

Where:
    YTM = yield to maturity
    n = frequency of the coupon payment

To calculate the approximate change in a bond's price for any change in yield:

Approx % change in bond price = (-)(Mod D)(change in yields) x 100

Where:
    Mod D = modified duration

Example:

The Macaulay duration for a 12 year semiannaul pay bond is 9.5.

a) If the yield to maturity is 10%, calculate the modified duration:

b) In your own words briefly describe what this modified duration means.

c) A 20 year semiannual pay bond has a Macaulay duration of 15.5 and a yield to maturity of 8%. If the yield falls to 7.25%, calculate the approximate percentage change in the bond's price:

Solutions:

a) Modified duration = Macaulay duration / [1 + (YTM/n)]

Modified duration = 9.5 / [1 + (10%/2)]

Modified duration = 9.0476

b) The modified duration tells us the approximate percentage change in the price of the bond. In this case, if yields change by 1%, the bond's price will change by approximately 9.0476%.

c) First we must calculate the Modified duration:

Modified duration = Macaulay duration / (1 + YTM/n) = 15.5 / (1 + 8%/2) = 14.9

Next calculate the yield change in decimal format:

7.25% - 8% = 0.75% which is 0.0075 in decimal format.

Now calculate the approximate percentage price change:

Approx % change in bond price = (-)(14.9)(-0.0075) x 100 = 11.175%

Knowledge of modified duration allows investors to create strategies to take advantage of expected interest rate movements.

Example:
How could an investor position her bond portfolio to take advantage of her expectation of a sharp increase in interest rates over the coming months?

Solution:
The investor would sell bonds that have large modified duration and buy bonds with short modified duration. This will reduce the volatility of her portfolio reducing the potential losses coming as a result of rising interest rates.

Exam tip:
Watch out on the exam they could say the investor should sell low coupon long maturity bonds and buy high coupon short maturity bonds, which of course are the properties of bonds with long and short durations!

Portfolio Duration
Portfolio duration is just the weighted average of the durations of all the bonds in the portfolio. Weightings are calculated based on the market values of the bonds, not the face values.

Note:
Portfolio duration is a reliable measure of volatility only for a parallel change in interest rates, that is, if the yield curve shifts up or down by the same amount at all maturities. It is not a reliable measure of volatility when interest rates change in a non-parallel way (ie. when the yield curve steepens or flattens), in this case we would need to look at how much each individual bond changes in value (but this is beyond the scope of the IMT exam!).

$$\text{Portfolio Duration} = \sum w \times D$$

Where:
  D = modified duration of the bond
  W = weighting of the bond in the portfolio = $\dfrac{\text{market value of the bond}}{\text{Portfolio market value}}$

Portfolio Duration Learning Example:
Use the following information to answer the next 3 questions

A fixed Income Portfolio Manager is reviewing the following bond portfolio:

Table 10

Bond	YTM	Market Value	Face Value	Duration
TLC Corp.	7.5%	$1,320,000	$1,000,000	1
SS Inc.	6.2%	$2,400,000	$2,000,000	3
HT Mfg.	3.3%	$1,280,000	$1,000,000	5
Total		$5,000,000	$4,000,000	

1.   **Calculate** the portfolio duration:

2.   If 5 year rates were to rise by 200 basis points and 1 year rates were to rise by only 80 basis points, the Portfolio Manager would be most likely to conclude:

   A.   the yield curve risk would be reflected on average by the portfolio duration
   B.   the increase in portfolio value would be equal to the bond durations times the rate change
   C.   the portfolio duration would not be a reliable measure of interest rate risk

3.   **Calculate** the price value of a basis point for the HT Mfg. bond:

Solutions:

1.   Use the market values, not the face values to calculate the weightings:

   Portfolio duration = 1 x (1,320/5,000) + 3 x (2,400/5,000) + 5 x (1,280/5,000) = 2.98

2.C   Portfolio duration is only applicable to parallel changes in interest rates.

3. $PVBP = (-)(D)(0.0001)(P_B) = (-)(5)(0.0001)(1,280,000) = -\$640$