Financial Math

For the CSC 1 and CSC 2 Exams, candidates are expected to have a high degree of understanding of time value of money principles, security valuation and basic statistics.

Formulas are not provided on the exam, therefore you must memorize them!

Candidates report that a lot of relatively easy marks are often missed because of a lack of familiarity with basic Math skills and most surprising – simply not knowing how to use a financial calculator correctly.

Current Yield - Stocks and Bonds

The current yield on a stock or bond is a measure that provides an investor with a quick and simple indication of the return potential for an investment.

The current yield is the income, dividend or interest, divided by the market price of the security. Candidates should realize that this is a one time yield measure as the market price of the security can change.

Current Yield (Dividend Yield) - Stocks

$$\text{Current Yield} = \frac{\text{Dividend}}{\text{Current Market Price}} \times 100$$

Example:

You buy one share of BCE for $33, BCE pays an annual dividend of $2.80 dividend?

What is the current dividend yield on the BCE shares?

$$\text{Dividend Yield} = \frac{\$2.80}{\$33} \times 100$$

$$= 8.48\%$$

Current Yield - Bonds

$$\text{Current Yield} = \frac{\text{Coupon in \$}}{\text{Current Market Price}} \times 100$$

Example:

Calculate the YTM for a 6 year annual pay 10% GOC bond purchased at 975.

What is the current yield on this bond?

$$CY = \frac{100}{975} \times 100$$

$$= 10.26\%$$

Example:

As a quick indication of the relative attractiveness of two potential investments you decide to calculate the current yield on ABC Inc. common stock and ABC Inc. 8% annual pay debentures. ABC common stock is currently trading at a price per share of $22.05 and pays an annual dividend of $1.80, while the debentures are currently valued at $980.

Calculate the current yield for both securities.

The answer is 8.163% for both!

As a follow up to the previous example, assume you have a middle aged, married client with a high tolerance for risk who is in the top marginal tax bracket, without doing any calculations, which investment would you select for your client? Why?

(Write your answer down and then email Prof. Gordon to see if your logic was correct)

Solution:
After tax return is the focus for taxable individual investors! Thus, you would select ABC common stock because the dividend income is taxed at a lower effective rate than the interest income from the debentures due to the dividend tax credit. Therefore, your client will have more money in his/her pocket after tax.

Exam Tip:
For individual investors, remember the focus should be on after tax real returns!

To calculate the after tax real return, use the following short cut formula:

$$ATRR = \frac{[\text{ Nominal Return} \times (1 - \text{Marginal Tax Rate})] - \text{Expected Inflation}}{(1 + \text{Expected Inflation})}$$

Yield to Maturity (YTM)

The YTM for a bond represents an expected return over the life of the bond assuming that all coupons are reinvested at that same YTM! This assumption is quite unrealistic, as interest rates are likely to change over time. (Link to reinvestment risk)

Bond dealers use YTM as a convenient way to determine the market value of a bond.

$$\text{Yield to Maturity} \quad = \quad \frac{\text{Coupon} + \dfrac{(1{,}000 - \text{Market Price}_B)}{n}}{(1{,}000 + \text{Market Price}_B) / 2}$$

Example:

Calculate the YTM for a 6 year annual pay 10% GOC bond purchased at 975.

$$\text{YTM} = \frac{100 + \dfrac{(1{,}000 - 975)}{6}}{(1{,}000 + 975) / 2}$$

Enter $100 $\boxed{\text{PMT}}$, 6 $\boxed{\text{n}}$, -975 $\boxed{\text{PV}}$, 1,000 $\boxed{\text{FV}}$

$= 10.55\%$

Solve for $\boxed{\text{I}}$ $= 10.58\%$

Market Value of a Coupon Paying Bond and the Market Value of a Strip Bond

The market value of a bond, or its price, is equal to the present value of the future coupon payments and the present value of the par or face value of the bond.

A Strip Bond has only one future cash flow, this could be the par value with no coupons (*P-strips*) or any one of the individual coupons (*C-strips*), thus to find the market value of a Strip Bond we simple calculate the present value of a lump-sum formula!

Bond Pricing

General Relationship: As Yields rise $\Rightarrow$ Bond Prices fall

Bond Price = PV (Cash Flows) + PV (Maturity Value) $\leftarrow$ this is the *"clean price"* of the bond.

Looks like our old friends PV of an annuity and PV of a lump sum!

Our suggestion is to calculate this in three steps:

Step 1: PV (Coupon Payments)
Present value of an annuity payment

END MODE:

$$PVA \quad = PMT \times \frac{1 - (1 + I)^{-n}}{I}$$

Enter the following information given in the question:

END

FV   n   I   PMT   solve for: PV

Step 2: PV (Maturity Value)
Present value of a lump sum payment

$$PV \quad = \frac{FV}{(1 + I)^n}$$

Enter the following information given in the question:

FV   n   I   PMT   solve for: PV

Step 3: Add your results in steps 1 and 2 together to get your Bond Price.
P_B = PV (Coupon Payments) + PV (Maturity Value)

Example:

Calculate the current market price of a 5 year annual pay 6.25% Government of Canada bond, if similar 5 year bonds are yielding 8%.

Step 1:

PVA $= 62.5 \times \dfrac{1 - (1.08)^{-5}}{0.08}$

    $= \$249.54$

Enter -$62.5 | PMT | , 5 | n | , 8 | I | , 0 | FV |

Solve for | PV | = $249.54

Step 2:

PV $= \dfrac{\$1,000}{(1.08)^5}$

    $= \$680.58$

Enter -$1,000 | FV | , 5 | n | , 8 | I | , 0 | PMT |

Solve for | PV | = $680.58

Step 3:

P_B = PV (Coupon Payments) + PV (Maturity Value) = $249.54 + $680.58 = $930.12

Additional Examples:

1.   Assume the Government of Canada issues a 20 year semi-annual pay bond with a coupon of 6%. A quick scan of the internet reveals, current market interest rates (or yields) for similar bonds are 7.5%.

   Calculate the bond's current market price

2.   Assume that market interest rates have changed and the bond is now valued at $1,020, c**alculate** the Yield to Maturity for this bond.

3.   Assume that an investment dealer strips off the coupons and principal to sell to investors as C-strips and a P-strip. Using the initial information as above, **calculate** the current market value of the P-strip.

4.   John and Mary Smith have a child attending the world famous Stampede School of the Arts, a private music academy located in Calgary Alberta. As part of the tuition, they are required to make a $25,000 donation to the academy's endowment fund. The payment is due 4 years from now. As the Smith's Financial Planner, which of the following investments would you recommend that would *best* fund this obligation insuring they have the required funds to make the donation?

   A   A $25,000 investment in the RBC Short-term Bond Fund.
   B   A Province of B.C. Strip Bond that matures in 4 years with a face value of $25,000
   C   A long-term Government of Canada 6% Bond with a face value of $25,000.
   D   A $25,000 investment in the RBC Long-term Bond fund.

Solutions:

1.   FV = 1,000, PMT = 60/2=30, I = 7.5/2=3.75, N=20x2=40, Solve for PV = 845.86
2.   FV = 1,000, PMT = 60/2=30, N=20x2=40, PV = -1,020, Solve for I = 2.914x2 = 5.829%
3.   FV = 1,000, PMT = 0, I = 7.5/2=3.75, N=20x2=40, Solve for PV = 229.337
4. B   The strip bond is the only choice that will provide the $25,000 with certainty.

Accrued Interest Calculation

Bonds pay interest on specific dates commonly referred to as the coupon payment date. When a bond is bought on any other date interest has built up (accrued) and is owed to the seller of the bond. The purchaser of the bond must pay the accrued interest as well as the purchase price of the bond to the seller. This is known as the *"full price"* or the *"dirty price"* of the bond. The price of the bond excluding the accrued interest is known as the *"clean price"*.

To calculate accrued interest you must know the coupon payment date, the settlement date, and the coupon rate. The coupon payment date and coupon rate will be given in the question information. You will need to figure out the correct settlement date. Use the following chart as a guide:

Bond	Settlement
Maturity of 3 years or less	T + 2 business days
Maturity over 3 years	T + 2 business days

Interest accrues from the first day following the last coupon payment date up to and including the settlement date.

Example:

Today is Tuesday February 6. Your client would like to invest $200,000 in a fixed income security. You have identified an 8% Government of Canada bond, due in 7 years on January 1, with a YTM of 8%, as a suitable investment.

(a)   **Calculate** the clean price of the bond.

(b)   **Calculate** the total amount of accrued interest and indicate which party will pay it and which party will receive it.

(c)   **Calculate** the full price of the bond.

There are over 50 pages in this workbook, full of demonstration learning examples designed to help you refresh and master the key financial concepts you need to know for both exam 1 and 2.

And remember…..as one of Prof. Gordon's students you are never studying alone….its like having your personal study coach standing ready to provide you with study support when you need it!

We hope you have a better idea of how this workbook can help add critical points to your exam score pushing you over the passing mark!

If you would like to access the remaining pages, just click on this link to complete the process.

Wishing you Exam Success!

Instructor/Author Profile:

Brian Y. Gordon, CFA, CFP, CIM, MBA, FCSI, is a tenured Professor in the School of Business at Centennial College in Toronto where he teaches Economics, Financial Accounting, Corporate Finance, the Canadian Securities Course, Personal Financial Planning and Investment Management.

Prof. Gordon is also a part-time faculty member at Concordia University in Montreal, where he teaches Economics and Investment Management courses at the MBA level.

Prof. Gordon has also lectured at Ryerson University in Toronto teaching Corporate Finance.

Since 1999, Prof. Gordon has been a featured lecturer and workshop facilitator for CFP® and CFA® review programs offered across Canada.

Prior to entering academia, Prof. Gordon developed his expertise in the discount brokerage, full service brokerage and banking industries, specializing in investment management, business development, strategic sales and marketing, and wealth management training.

Prof. Gordon holds a BA in Economics from the University of Toronto, an MBA from Heriot-Watt University in the UK, and was awarded his CFA charter in 1999. In 1995, Prof. Gordon was granted a fellowship from the Canadian Securities Institute, earning the prestigious FCSI designation.

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