Pricing and Valuation of Forward Commitments

Professor's Comment:
This reading has only four learning outcome statements, but don't be fooled into thinking it is something you can skip. I think you must familiarize yourself with the terminology and notation used in the examples, because I think that is what you are going to see if it shows up on the exam.

Introduction
Futures, forwards and swaps are known as **contracts of commitment** because no matter what the outcome is, both parties must take action – they are locked-in!

Pricing and Valuation
The reading focuses on the perspective of an ***arbitrageur*** and I think you should be ready to answer qualitative questions about the steps involved in arbitrage and the following basic concept:

> The arbitrageur's perspective means that we only enter into transactions where:
>
> 1) ***we do not use our own money*** (we borrow or lend) and
> 2) ***we do not take on any price risk*** (use off-setting buy/sell positions)

The no arbitrage approach to pricing and valuation is based on the following assumptions:

1) Replication is possible
2) No transaction or liquidity costs (ie. markets are frictionless)
3) Short selling is permitted
4) We can borrow/lend at the risk free rate

The text also points out the ***"law of one price"***, which says, in the context of derivative investments, if two investments have the same future cash flows, then they **MUST** have the same current price.

Pricing refers to finding the forward price or fixed rate that will set the initial value at time (T = 0) equal to zero.

Value refers to finding the present value of the difference in cash flows for two positions (ie. long side and short side, fixed side and floating side).

Exam Tip:
The reading uses tables to display cash flows, so I think you need to look at the "blue box examples" so you become familiar with the notation.

Pricing and Valuing General Concepts

The basic relationship between the spot and forward price is:

$$F = S_0\,(1 + R_F)^T$$

Where:

F = Forward price

S_0 = Spot price

R_F = Risk free rate

T = Time to maturity (expiration)

Key Points about Price:

- You can think of forward price and futures price as being exactly the same…meaning you can use the formula above for either forwards or futures contracts….so don't get confused about capital "Fs" or small "fs"….I will point out the differences you need to know as we go along.

- The forward price does not reflect expectations of future underlying asset price, only interest rates and time affect the forward price.

Exam Tip:

On the exam watch for statements like, "…an asset's price is expected to increase substantially in the future, so the forward price should increase…" NO, NO, NO this is not true!!

Valuation

At initiation of the contract, we would expect the value of the contract to be zero ($V_0 = 0$).

At expiration of the contract, the value of the contract is equal to the difference between the spot and forward price:

<u>Value from the long side's perspective:</u> $V_T = S_T – F$

<u>Value from the short side's perspective:</u> $V_T = F – S_T$

During the life of the contract, the value of a contract is based on the difference between the present value of the initial (F_{t-}) and new (F_t) forward prices or (equivalently) the difference between the spot price (S_t) and the present value of the initial forward price.

<u>Value from the long side's perspective:</u> $V_t = (F_t – F_{t-})/(1 + R_F)^t$ or $V_t = S_t – F/(1 + R_F)^t$

<u>Value from the short side's perspective:</u> $V_t = (F_{t-} – F_t)\,/(1 + R_F)^t$ or $V_t = F/(1 + R_F)^t – S_t$

Special Note for Futures Contracts (which require daily settlement)

During the life of the contract, the value of the contract, **before mark to market**, is equal to the difference between the initial futures price and new futures price:

Value from the long side's perspective: $V_t = f_{new} - f_{old}$

Value from the short side's perspective: $V_t = f_{old} - f_{new}$

During the life of the contract, the value of the contract, **after mark to market**, is equal to zero.

Bottom Line:
On a trivial note, if you are asked, during the life of the contract, you would expect a futures contract to have a lower value than a similar forward contract.

Quick Example:
A trader entered into a long forward position, expiring in 6 months, at a price of $60. Three months later the underlying is trading at $63. If the risk free rate is 3%:

a) **calculate** the new forward price

b) **calculate** the value of the trader's contract

Solutions:

a) $F_t = S_t(1 + R_F)^t = \$63(1.03)^{0.25} = \$63.467$

b) $V_t = (F_t - F_{t\text{-}}) / (1 + R_F)^t = (\$63.467 - \$60) / (1.03)^{0.25} = \$3.44$

Carry Arbitrage and Reverse Carry Arbitrage Models

Professor's Comment:
I think you must be familiar with the steps in these models and be ready to recognize which to use and what actions (steps) to take in order to take advantage of an arbitrage opportunity….its a no brainer for an exam question!

Basic relationships:

- If $F = S_0 (1 + R_F)^T$ then no arbitrage opportunity exists
- If $F > S_0 (1 + R_F)^T$ then do carry arbitrage
- If $F < S_0 (1 + R_F)^T$ then do reverse carry arbitrage

Carry arbitrage means, the forward is relatively overpriced compared to the spot and carry costs, so you buy the asset (at the spot) and hold it (carry it) and immediately sell the asset forward for delivery at maturity….of course you do this with borrowed funds so that we satisfy the two criteria previously stated (not our money and no price risk).

Reverse carry arbitrage means, the forward is relatively underpriced compared to the spot and carry costs, so you short sell the asset (at the spot) and immediately buy the asset forward for delivery at maturity….you lend out the proceeds from the short sale so that we satisfy the two criteria previously stated (not our money and no price risk).

Exam tip:
On the exam, look for the following "steps" (actually they are done at the same time!):

Carry Arbitrage: 1) Sell forward, 2) Buy spot, 3) Borrow purchase price, and 4) Borrow present value of the arbitrage profit.

Reverse Carry Arbitrage: 1) Buy forward, 2) Short sell spot, 3) Lend proceeds, and 4) Borrow present value of the arbitrage profit.

Extending the Simple Pricing Model to include Costs and Benefits
Up to this point we have been talking about financial assets that do not have cash flows or incur any associated costs. The text now talks about assets that have ***carry benefits***, such as dividends and coupon payments, and ***carry costs*** such as waste, storage and insurance.

So we adjust our basic relationship between the spot and forward price to:

$$F = S_0 (1 + R_F)^T + FV(Costs) – FV(Benefits)$$

Where:
- F = Forward price
- S_0 = Spot price
- R_F = Risk free rate
- T = Time to maturity (expiration)
- FV(Costs) = Future value (at expiration) of the carry costs
- FV (Benefits) = Future value (at expiration) of the carry benefits

The text tells us that carry costs are relate to commodities and we do not discuss commodities in this reading, so we can ignore them and thus simplify our formula to:

$$F = S_0 (1 + R_F)^T – FV(Benefits)$$

Key Points to Know:
- Higher benefits lead to lower cost of carry and a lower forward price.
- Steps in carry arbitrage now become, 1) Sell forward, 2) Buy spot, 3) Reinvest cash flows at R_F, 4) Borrow purchase price, and 5) Borrow present value of the arbitrage profit.
- Individual dividend paying stock use simple annual compounding, while stock indexes use continuous compounding.

Quick Example:
A trader is looking to price a futures contract on an asset that expires in nine months. The asset is currently prices at $33 and the risk free rate of 6%. Based on this information, **calculate** the futures price if the underlying asset's storage costs at expiration equal $2.10 and the compound value at the time of the futures expiration of the positive cash flow from the underlying asset is $0.55.

Solution:
$F = S_0(1 + R_F)^T + \text{Storage Costs} – \text{Cash Flows} = \$33(1.06)^{0.75} + \$2.10 – \$0.55 = \$36.02$

Interest Rate Forwards and Futures

Spot Market – Borrowing/Lending Today
The text begins with a quick discussion of **LIBOR**, which we know is a floating rate of interest and **Eurodollar time deposits,** which are agreements to borrow (or lend) with the interest calculated on an ***"add on basis"***.

> Terminal Amount = Notional Amount [1 + L(ACT/360)]
>
> Where:
> L = LIBOR rate (given as an annual rate)
> ACT = Number of days in the time deposit

Forward Market – Forward Rate Agreements (FRAs)
FRAs are forward agreements, based on LIBOR, to lock-in a rate to borrow or lend. The long party to an FRA, known as the ***"receive floating"*** side, is the borrower and the short side, known as the ***"receive fixed"*** side, is the lender.

Key Points to Know:

- The receive floating side (or borrowers) use FRAs to protect themselves against an increase in interest rates.
- The receive floating side gains when interest rates increase and lose when interest rates fall.
- The receive fixed side (or lenders) use FRAs to protect themselves against a decrease in interest rates.
- The receive fixed side gains when interest rates fall and lose when interest rates increase.
- The ***price*** of an FRA is the fixed rate that sets the initial value (at T= 0) to zero. ….to be technical, it is the ***implied forward rate***!
- FRA payments are based on a 30/360 day count.
- FRAs pay based on ***"advanced set, advanced settled"*** basis.

FRA Notation and Interpretation

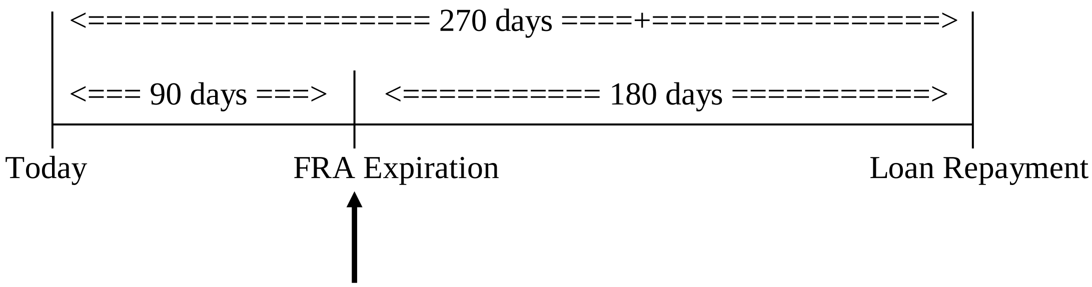
Notation	Contract Expires in	Underlying Rate
1 x 3	1 month	60 day LIBOR
3 x 6	3 months	90 day LIBOR
6 x 12	6 months	180 day LIBOR

The first number refers to the expiration of the agreement and the second number refers to the time until the loan matures and the difference between these two numbers refers to the underlying rate.

Payment Example:

The following data is for an FRA 3 x 9 (an FRA expiring in 90 days for which the underlying is 180 day LIBOR):

- Forward contract rate as quoted by a dealer is 5.5%
- Notional Principal is $10,000,000
- Assume at expiration the rate on 180 day LIBOR is 6%



Payment to settle the FRA at expiration (received by the "receive floating" party):

$$\$10{,}000{,}000 \left(\frac{(0.06-0.055)(180/360)}{1 + 0.06(180/360)} \right) = \$24{,}272$$

Difference between actual rate that exists at expiration and agreed upon rate

Adjustment (or ***discount rate for the FRA***) to reflect the fact that payment is ***"advanced settled"***.

The ***value of a FRA*** equals the net of the present value of the cash flows paid to each party. The value is positive to the winner and negative to the loser….remember if interest rates rise, the long side is a winner and the short side is a loser and if interest rates fall, the long side is the loser and the short side is the winner.

FRA Price

Remember we are trying to find the fixed rate that sets the initial value (V_0) equal to zero. We do this by calculating the implied forward rate:

$$\text{Rate} = \left(\frac{1 + (\text{LIBOR to Maturity})(m/360)}{1 + (\text{LIBOR to Expiration})(n/360)} - 1 \right)\left(\frac{360}{\#} \right)$$

Where: m = number of days to maturity
n = number of days to the expiration of the FRA
# = number of days in the loan period

<u>FRA Pricing Example:</u>

Logan Corp. needs to borrow $5,000,000 for 6 months starting 2 months from today. The CFO is concerned that interest rates may move in an adverse way over the next 2 months and decides to use an FRA to reduce the risk. The current LIBOR term structure is given below in Table 1.

Table 1

60 day LIBOR	3%
180 day LIBOR	3.5%
240 day LIBOR	4%

1) **Briefly describe** how the CFO would use a FRA to accomplish the risk reduction strategy?

2) **Briefly describe** the FRA structure that would reduce the risk faced by Logan Corp.

3) Based on the information given, **calculate** the price of the appropriate FRA:

Solutions:

1) The CFO should enter into a long position (or the receive floating side) in a FRA.

2) The FRA structure would be a 2 x 8.

3)

$$\text{Rate} = \left(\frac{1 + (4\%)(240/360)}{1 + (3\%)(60/360)} - 1 \right) \left(\frac{360}{180} \right) = 0.04312 \text{ or } 4.312\%$$

Valuing FRAs
To find the value, we price a new FRA and then just like any forward, we calculate the difference between the present value of the new FRA and old FRA price.

Continuing with the previous example…..
Assume the CFO entered into the FRA and 30 days have passed. The current LIBOR term structure is given in Table 2.

Table 2

30 day LIBOR	3.3%
210 day LIBOR	4.5%
330 day LIBOR	4.9%

1) **Calculate** the value of the FRA:

Solution:

Step 1, price a new FRA:

$$\text{Rate} = \left(\frac{1 + (4.5\%)(210/360)}{1 + (3.3\%)(30/360)} - 1 \right) \frac{360}{180} = 0.04687 \text{ or } 4.687\%$$

Step 2, calculate the value of the FRA:

$$V_t = \frac{(4.687\% - 4.312\%)(180/360)(\$5{,}000{,}000)}{1 + (4.5\%)(210/360)} = \$9{,}135.20$$

Exam tip:
Watch out for rounding errors…the text uses 3 decimal places and you should as well.