Hi and welcome,

This is a brief sample to provide you with an idea of what my CFA study notes look like.

The style is to provide key content coverage followed by working examples.

Mixed in you will see my *"Professor's Comments"* and *"Exam Tips"*, I've included these to give you my thoughts on how I believe something will show up on the exam or how best to approach learning a concept….or even if I believe you should just skip something altogether.

If you have written the exam before, you can use my materials to get back up to speed very quickly.

If you are a first time writer, you can use my materials to help focus your study efforts and build your knowledge base.

If you have any questions, please do not hesitate to reach out to me by email.

All the best,

Prof Brian Gordon, CFA, CFP, CIM, MBA, FCSI
Director of Learning
Exam Success

Level 1

Quantitative Methods (QM)

READING 6:
The Time Value of Money

READING 7:
Statistical Concepts and Market Returns

READING 8:
Probability Concepts

If you can handle the follow items from Quants you should be in good shape for the exam.

1. Calculate EAR given stated annual interest rate

2. Calculate PV and FV for Annuities and Lump Sums and Perpetuities

3. Calculate relative and cumulative frequencies

4. Comment on dataset presented

5. Calculate mean, Geometric Mean, harmonic mean, median and mode

6. Calculate range, mean absolute

7. Calculate standard deviation for a sample

8. Use Chebyshev

9. Calculate CV and Sharpe

10. Comment on skewness

11. Calculate covariance and correlation using probabilities

12. Calculate standard deviation using probability

13. Calculate standard deviation and expected return for a portfolio

14. Use Bayes formula

Time Value of Money

Professor's Comment:
The purpose of this reading is to establish a basic foundation in the calculation of present value and future values. If you have a strong math background, I think you can just quickly skim the reading. This is the math which underlies the valuation of securities.

The main idea to take away is the value of any security is equal to the present value of its future cash flows.

Discount Rates, Interest Rates and Opportunity Cost

Cash flows that occur at a future date can be discounted to provide a present value and present values can be grown out to future values by applying an interest rate.

When we are forced to make a decision, rational decision makers look to maximize their wealth (or utility). An interest rate can provide insight into finding the best available choice (the one that maximizes utility)….and the next best choice that we forego is referred to as the opportunity cost of the decision.

Interest Rate Components

The text tells us that an interest rate is composed of the real rate, expectations for inflation and compensation for risk premiums. In formula format:

$$r = \text{Real risk free rate} + \text{expected inflation} + \text{specific risk premium}$$

Other Definitions to know:

Stated annual rate (r) – this is the nominal rate and it is usually given in the question.

Periodic rate (r/n) – this is the stated annual rate adjusted for the compounding frequency (also known as the Holding Period Return).

Quick Example:

Given a stated annual rate of 10%, **calculate** the quarterly holding period return:

Solution:

HPR (or periodic return) = r/n = 10% / 4 = 2.5%

Effective Annual Rate

Holding period returns (*or periodic returns*) are often converted into annual figures for comparison purposes. <u>Be aware</u> that holding periods can be less than (or greater than) one year.

For periods less than one year use the following formula:

> **Effective annual rate** $= (1 + r/n)^n - 1$
>
> Where:
> > r is the stated annual rate
> > n is the frequency of compounding

For periods greater than one year use the following formula:

> **Effective annual rate** $= (1 + HPR)^{m/h} - 1$
>
> Where:
> > HPR is the holding period return (or periodic return)
> > m is the annual period expressed in years, months or days
> > h is the given holding period in years, months or days

Effective Annual Rate Quick Examples:

1. The effective annual rate for an 8% stated annual return compounded semiannually would be *closest to*:

    A.   4%
    B.   8.16%
    C.   16%

2. The effective annual rate for an 18 month periodic return of 20% would be *closest to*:

    A.   13%
    B.   13.5%
    C.   22%

Solutions:

1. B   Effective annual return $= (1 + 8\%/2)^2 - 1 = 0.0816$ or 8.16% %

2. A   Since we are given the holding period in months, the annual period "m" is 12…because there are 12 months in one year.

   Effective annual return (or annualized return) $= (1 + 20\%)^{12/18} - 1 = 0.1292$ or 12.92%

TVM Examples:

1. As of January 1, 2019, an annuity offers $5,000 per year for seven years with the first payment due January 1, 2024. If the annual interest rate is 11.5%, what is the present value of the annuity?

   A. $13,453
   B. $23,185
   C. $15,000

2. An investor deposits $50,000 in a tax exempt account that earns 6% compounded on a monthly basis. After 5 years the value of the account would be *closest to*:

   A. $53,084
   B. $66,911
   C. $67,442

3. What is the present value of a ten-payment annuity of $1,000 at 8% interest?

   A. $5,002
   B. $6,710
   C. $7,247

4. Three years ago an investor's portfolio was worth $55,000. If it is worth $92,000 today, then the compound annual growth rate is closest to:

   A. 67.2%
   B. 55.7%
   C. 18.7%

Solutions:

1.C First find value of the annuity as of January 1, 2024:
BGN, FV = 0, I = 11.5, N = 7, PMT = 5,000, CPT PV = 25,851.47

Now find the value as of January 1, 2019:
FV = 25,851.47, I = 11.5, N = 5, PMT = 0, CPT PV = 15,000.68

2.A FV = PV $(1 + r/n)^{(m)(n)}$ = $50,000$(1 + 6\%/12)^{(5 \times 12)}$ = $67,442.51

3.B END, FV = 0, I = 8, N= 10, PMT = 1,000, CPT PV = 6,710.08

4.C g = [End Value/Beg Value]$^{1/n}$ − 1 = [92,000/55,000]$^{1/3}$ − 1 = 0.187 or 18.7%

Statistical Concepts and Market Returns

The overall theme of this reading is that we can organize data to draw conclusions and then make decisions.

Descriptive statistics involves the collection and summary of data

Inferential statistics focuses on using the data to learn about the population from which the sample has been drawn.

Nominal scales categorize data but do not rank them. They provide the least useful amount of information to draw conclusions from.

Ordinal measurements describe order, but not relative size or degree of difference between the items measured.   In this scale type, the numbers assigned to objects or events represent the rank order (1st, 2nd, 3rd, etc.) of the entities assessed. (ie. credit ratings)

Interval scales tell us about the order of data points, and the size of the intervals in between data points.

A **ratio scale** is an interval scale with a true zero point, which permits higher level of comparisons and interpretation of the magnitude of the differences. (ie. rates of return)

Bottom Line:
As financial analysts we want to apply the statistical concepts to our valuation models to provide flexibility in our predictions of security prices.

Frequency Distribution
A frequency distribution is a tabular display of data summarized into a relatively small number of intervals.

A **histogram** is a bar chart of data that have been grouped into a frequency distribution

A **frequency polygon** plots the midpoint of each interval on the x-axis and the absolute frequency on the y-axis and then connects each point with a continuous line.

The **Cumulative Frequency** is equal to the sum of the absolute frequencies

The **Relative Frequency** is equal to the absolute frequency divided by the number of data points

The **Cumulative Relative Frequency** is equal to the cumulative frequency divided by the number of data points

LINK: keep this in mind when you are reading about distributions in Study Session 3!

Frequency Distribution Example:

An analyst observes the following holding period returns (%) for client portfolios:

-9, -6, -5, -3, -1, 0, 1, 3, 4, 7, 10, 15, 17, 18, 21

1)   **Calculate** the range:

2)   **Calculate** the interval width assuming the analyst divides the data into 5 intervals:

3)   Based on your answers to the previous questions, **determine** the intervals:

4)   Based on your answers to the previous questions **complete** the following table:

Interval	Beginning Point	Data Points	End Point	Absolute Frequency
1				
2				
3				
4				
5				

5)   Which interval has the smallest relative frequency?

6)   **Calculate** the cumulative relative frequency of the third interval:

Solutions:

1) Range = highest – lowest = 21 – (-9) = 30

2) Interval width = Range / k = 30 / 5 = 6

3) To find the intervals add the interval width to the lowest value and repeat until all values are included in an interval:

Interval 1: –9 + 6 = –3
Interval 2: –3 + 6 = 3
Interval 3: 3 + 6 = 9
Interval 4: 9 + 6 = 15
Interval 5: 15 + 6 = 21

Note: intervals do not overlap and data points fit into only one interval!

4)

Interval	Beginning Point		Data Points		End Point	Absolute Frequency
1	–9	$\leq$	–9, –6, –5	$<$	–3	3
2	–3	$\leq$	–3, –1, 0, 1	$<$	3	4
3	3	$\leq$	3, 4, 7	$<$	9	3
4	9	$\leq$	10	$<$	15	1
5	15	$\leq$	15, 17, 18, 21	$\leq$	21	4

5) Relative frequency = Absolute frequency / N

Interval 1: Relative frequency = 3 / 15 = 0.2 or 20%
Interval 2: Relative frequency = 4 / 15 = 0.267 or 26.67%
Interval 3: Relative frequency = 3 / 15 = 0.2 or 20%
Interval 4: Relative frequency = 1 / 15 = 0.0667 or 6.67%   <= smallest!
Interval 5: Relative frequency = 4 / 15 = 0.267 or 26.67%

6) Cumulative relative frequency = $\sum$ absolute frequencies / N

For the third interval:

Cumulative relative frequency = (3 + 4 + 3) / 15 = 0.667 or 66.67%